cylindrical quartz irradiation tubes (~0.75 mm i.d.) contained an irradiated volume of 1 ml. Solutions were degassed by the freeze–pump–thaw cycle. Irradiation times were chosen so that the degree of conversion never exceeded 5%. Quantitative analyses were made on a Hewlett-Packard Model 5750 gas chromatograph fitted with a polypropylene glycol column (12 ft, UCON-50LB-550X, 20%) at 160°.

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The Optical Rotation of α-Bromopropionitrile and 3-Methyl-5-bromo-1-cyanoadamantane from Polarizability Theory

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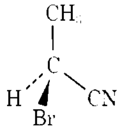
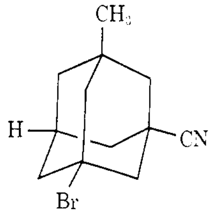
Abstract: The optical rotations of α-bromopropionitrile (BPN) and 3-methyl-5-bromo-1-cyanoadamantane (MBCA) at 5893 Å are calculated from polarizability theory, assigning isotropic polarizabilities to all atoms and, alternatively, single isotropic or anisotropic polarizabilities to the CN group. The calculations make use of (i) modified polarizability data based on the dipole interaction model and (ii) the exact calculation of the relay tensors. These modifications lead to a much better agreement with experiment than earlier calculations for both compounds. The isotropic atom model for CN gives intrinsic molar rotations of L-BPN in the range −55 to −74 deg cm²/dmol (experimental value −19) and of MBCA of the same configuration in the range −0.29 to −0.94 (experimental value −0.82 for unknown configuration). The other models for CN show ambiguity of sign for BPN but not for MBCA. Displacement of the N atom in a manner approximating electrostatic repulsion by Br reverses the sign of rotation for MBCA but not for BPN. Rotation of the CH_3 group produces significant changes in optical rotation, but the thermal average optical rotation assuming reasonable energy barriers is not significantly different from the rotation of the staggered form.

A previous attempt[1] to account for the optical rotations of α-bromopropionitrile (BPN) and 3-methyl-5-bromo-1-cyanoadamantane (MBCA) by means of a polarizability theory which included all atoms was unsatisfactory for two reasons that were evident at the time: (i) the sign of the rotation calculated for BPN was inconsistent with the known absolute configuration, and (ii) the perturbation series used to calculate the rotations did not converge rapidly. In a subsequent study[2] it was found that the additive atom polarizabilities used in these calculations were invalid for calculating polarizability interactions, and a more valid set of atom polarizabilities and CN group polarizabilities[3] was determined. In addition, it has been found[4,5] that the perturbation approximation can be avoided by using an exact treatment of dipole–dipole interactions to calculate the set of "relay tensors" which determine both the optical rotation and molecular polarizability. The object of this paper is to show how these developments affect the rotations calculated for BPN and MBCA.



L-BPN



MBCA

We are confined in this study to calculations at the sodium D line, 5893 Å, since our polarizability data apply only at this wavelength. However, the calculations serve as a more realistic test of the polarizability theory than previous attempts because of the modifications noted above. While we do not obtain information on rotatory dispersion, the present theory treats intramolecular interactions which are generally omitted in the quantum mechanical calculations of rotatory dispersion. This approach should therefore serve as a useful supplement to quantum mechanical methods in general and particularly in cases such as the present ones, where the available information needed for a quantum mechanical calculation is very limited.

The molecules studied here serve as useful test cases because they have simple, essentially unstrained structures, and because their lack of conformational freedom minimizes the need for conformational averaging. In addition, the L configuration of BPN has been identified as the levorotatory form,[1,6] though the configuration of MBCA has not been related to sign of rotation.

Calculations

The molecule is regarded as being made up of N units which interact by means of the fields of their

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(1) J. Applequist, P. Rivers, and D. E. Applequist, *J. Amer. Chem. Soc.*, **91**, 5705 (1969).

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(3) J. Applequist and J. R. Carl, *J. Phys. Chem.*, 77, 2090 (1973).

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(6) K. L. Berry and J. M. Sturtevant, *J. Amer. Chem. Soc.*, **63**, 2679 (1941).

induced dipoles. The intrinsic molar rotation $[m]$ at vacuum wavelength λ and the mean molecular polarizability $\bar{\alpha}$ are calculated by means of the equations[2,5,7]

$$[m] = (48\pi^2 N_0/\lambda^2)\sum_{i=1}^{N-1}\sum_{j=i+1}^{N}\mathbf{r}_{ij}\cdot\mathbf{b}_{ij} \qquad (1)$$

$$\bar{\alpha} = \frac{1}{3}\mathrm{Tr}\sum_{i=1}^{N}\sum_{j=1}^{N}\mathbf{B}_{ij} \qquad (2)$$

where $\mathbf{r}_{ij}$ is the vector from unit i to unit j, $\mathbf{B}_{ij}$ is the relay tensor connecting the dipole moment of unit i to the field at unit j, and $\mathbf{b}_{ij} = \mathbf{B}_{ij}:\boldsymbol{\varepsilon}$, where $\boldsymbol{\varepsilon}$ is the third order permutation tensor. N_0 is Avogadro's number and Tr stands for "trace." In the present work the $\mathbf{B}_{ij}$ are calculated by the exact method,[2,5] unless otherwise noted.

The present calculations are based on the same atomic coordinates as our earlier calculations,[1] and all atoms are included. The assumed absolute configurations correspond to the L configuration of BPN. The H, Br, and aliphatic C atoms are regarded as separate isotropic units with the following polarizabilities derived[2] from molecular polarizabilities at 5893 Å in terms of the interaction model (Å^3): H, 0.135; C, 0.878; Br, 2.88. Three alternative models are adopted for the CN group, as in our treatment[3] of the polarizability of nitriles.

Model I. The C and N atoms are treated as separate isotropic units with polarizabilities α_C and α_N in the range of the optimum values found to fit mean polarizability data.[2]

Model II. The CN group is treated as a single isotropic unit whose polarizability α_{CN} and distance from the C atom to which it is attached (the "C–CN distance") are in or near the optimum range found for these parameters.[3]

Model III. The CN group is treated as a single anisotropic unit located at the midpoint of the CN bond. The mean polarizability $\bar{\alpha}_{CN}$ and anisotropy δ_{CN} are taken in or near the optimum range found for these parameters.[3]

The need for this special treatment of the CN group arises from the finding[3] that none of these models appears to be uniquely suitable for representing this group in treating the molecular polarizability and that the optimum parameters for each model lie within a fairly extended range. Thus, it is of interest to learn whether the optical rotation is similarly insensitive to these parameters or to the choice of model.

Table I gives the calculated $[m]_D$ and $\bar{\alpha}$ for selected sets of the CN parameters covering the optimum ranges. The parameters given in parentheses were slightly outside the optimum range for mean polarizabilities of nitriles but were in a range which gave a somewhat improved fit to molecular anisotropy data. In all of the calculations for Table I, the methyl group was placed in a staggered conformation; *i.e.*, the torsion angle[8] θ (C,C) was taken as 60°.

Table I. Calculated Optical Rotation and Polarizability of BPN and MBCA at 5893 Å

Model I		BPN		MBCA	
α_C, Å^3	α_N, Å^3	$[m]_D$, deg cm^2/dmol	$\bar{\alpha}$, Å^3	$[m]_D$, deg cm^2/dmol	$\bar{\alpha}$, Å^3
0.22	0.85	−54.7	9.36	−0.937	23.41
0.36	0.52	−73.7	9.38	−0.839	23.43
0.75	0.11	−67.9	9.15	−0.291	23.15
Model II					
α_{CN}, Å^3	C–CN, Å				
2.00	2.00	+16.1	9.32	−1.19	23.33
1.82	1.82	−0.64	9.32	−1.22	23.35
1.74	1.78	−5.6	9.28	−1.23	23.31
(1.58)	(1.69)	−19.9	9.27	−1.10	23.30
Model III					
$\bar{\alpha}_{CN}$, Å^3	δ_{CN}				
2.45	−0.60	+73.7	9.24	−1.89	23.11
(2.05)	(−0.05)	+22.0	9.31	−1.22	23.31
(1.95)	(+0.12)	+9.6	9.33	−1.05	23.36
(1.75)	(+0.50)	−13.8	9.38	−0.73	23.46
	Exptl[a]	−19[b]	9.41[b]	−0.820[c]	23.44[d]

[a] The experimental $[m]_D$ values differ from those in ref 1 because the latter include an erroneous factor of $6/(n^2 + 5)$. See discussion in ref 5. [b] Reference 6. [c] Reference 1. Absolute configuration of experimental compound is not necessarily that assumed in the calculations. [d] Sum of additive atom polarizabilities.[2]

In the case of MBCA, the sign and magnitude of $[m]_D$ are relatively insensitive to the CN parameters and are in fair agreement with experiment for the compound of unknown absolute configuration. For BPN the sign is insensitive to CN parameters only for model I. For models II and III the correct sign and rough magnitude are found only in a limited range of the CN parameters. This same range gave improved values of the molecular anisotropies but was not quite optimal for the mean polarizabilities.[3] The $\bar{\alpha}$ values are quite insensitive to CN parameters near the optimum range and are in fair agreement with experimental data, as was the case with other nitriles studied.[2,3]

Effects of Atom Displacements. Since rigid structures were assumed in the above calculations, it is desirable to explore the effects of some reasonable deviations from these idealized structures. Such effects were considered also in our earlier calculations.[1] First, we approximate the effect of electrostatic repulsion between the electronegative Br and N atoms by displacing the N atom perpendicular to the plane defined by the C–CN bond and the C–CH$_3$ bond in the direction which increases the separation between N and Br. For MBCA it is possible to reverse the sign of the calculated rotation by a relatively small displacement of N; *e.g.*, a 0.023 Å displacement gives $[m]_D$ = +0.80 deg cm^2/dmol, using model I for CN with α_C = 0.36 and α_N = 0.52 (*cf.* Table I). For BPN the same displacement gives $[m]_D$ = −81; *i.e.*, a distortion of this type does not tend to reverse the sign of the rotation. In the case of MBCA this effect is rather critical, since it means that the sign of the calculated rotation is sensitive to structural information which is not known precisely.

Second, we consider the effect of rotation of the CH$_3$ group. Figures 1 and 2 show the dependence of $[m]_D$ on the torsion angle θ (C,C) for BPN and MBCA, treating the CN group by model I with α_C = 0.36 and α_N = 0.52. For BPN the sign of $[m]_D$ is not affected by internal rotation, and the amplitude of the variation is similar to that calculated for ethane (also shown in Figure 1), though the dependence on torsion angle is

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(7) Equation 1 is smaller by a factor of 2 than the corresponding equation in ref 1, for reasons discussed in ref 5.

(8) IUPAC-IUB Commission on Biochemical Nomenclature, *Biochemistry*, **9**, 3471 (1970).

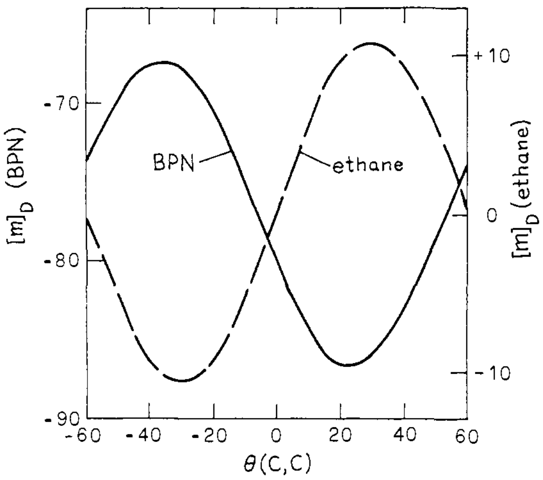

Figure 1. Intrinsic molar rotation of BPN and ethane (deg cm²/dmol) as a function of torsion angle for CH_3 rotation, calculated for isotropic atom model.

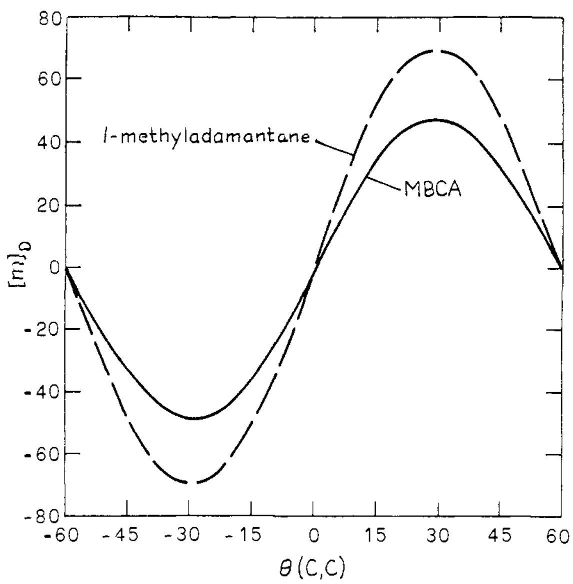

Figure 2. Intrinsic molar rotation of MBCA and 1-methyladamantane (deg cm²/dmol) as a function of torsion angle for CH_3 rotation, calculated for isotropic atom model.

Table II. Comparison of Third Order and Exact Rotations of BPN and MBCA[a]

Compd	Method	$[m]_D$ Additive polarizabilities[b]	 Interaction polarizabilities
BPN	3rd order	+2.12	+0.88
BPN	exact	+5005.	−73.7
MBCA	3rd order	−0.0406	−0.0096
MBCA	exact	−3508.	−0.839

[a] The CN group is treated as in model I, with additive polarizabilities as in ref 1 and interaction values $\alpha_C = 0.36$ and $\alpha_N = 0.52$. [b] The third order calculations in ref 1 are larger due to a factor of 2 error in the equation used in that work.[5]

quite different for the two. For MBCA the sign and magnitude of $[m]_D$ are strongly dependent on torsion angle, in a manner very similar to that calculated for 1-methyladamantane. The rotation of the latter vanishes due to symmetry at θ (C,C) = 0 and ±60°, while for MBCA $[m]_D$ is −0.933 at 0° and −0.839 at ±60 (Table I). A thermal average $[m]_D$ of −0.850 is calculated for MBCA at a temperature of 25°, assuming a 3-kcal sinusoidal energy barrier with minima at θ (C,C) = ±60°. The temperature coefficient $d[m]_D/dT$ is calculated to be -4×10^{-5}, in qualitative agreement with the very small temperature dependence observed[1] between 24 and 46°. The very slight effects of CH_3 rotation predicted in this way depend on the assumption that the energy barrier is symmetrical about the staggered position, since the rotation is changing rapidly and reverses sign in the vicinity of this position. An attempt was made to see whether electrostatic interactions among the bridgehead substituents in MBCA would change the barrier sufficiently to affect these results. Monopole charges were assigned on the basis of known group dipole moments to the bridgehead and methyl C, H, and Br atoms and the CN group, and the Coulombic energy was added to the assumed 3-kcal barrier. The effects on the calculated average rotation were negligible, due to the fact that the Coulombic energy barrier was only about 0.006 kcal. Moreover, the Coulombic barrier was unsymmetrical about the staggered position only when the N atom was displaced as described above, and even for N displacements as large as 0.2 Å the dissymmetry introduced was not sufficient to change the thermal average rotation appreciably from the rotation of the staggered form. Thus, it appears unlikely that any further ambiguity in the sign of $[m]_D$ of MBCA could arise from the effects of CH_3 rotation.

Comparison with Perturbation Calculations. The present calculations appear to be more realistic and in better agreement with experiment than our earlier calculations[1] for two reasons: (i) the additive atom polarizabilities used earlier have been replaced by values obtained from an interaction model, and (ii) the perturbation treatment of atom interactions has been replaced by an exact treatment. Table II is given here to emphasize the importance of these modifications. The perturbation and exact calculations are compared using both the additive and interaction atom polarizabilities. The third order perturbation term is given, since this is the lowest order term which does not vanish for isotropic atoms. The discrepancies among the various calculations are enormous, and only the exact calculations using interaction polarizabilities show reasonable agreement with experiment (*cf.* Table I).

Discussion

The polarizability model has been reasonably successful in the present calculations. For BPN the correct sign and order of magnitude of rotation are obtained, and the calculations are not overly sensitive to the atom displacements considered here. For MBCA the correct magnitude is obtained over an even greater range of parameters than for BPN, but since we do not know the absolute configuration of the experimental compound, and since the sign is sensitive to the displacement of N, the significance of this result is not clear. However, there appears to be little doubt that the rotation of both compounds arises primarily from the polarizability mechanism.

The isotropic atom model used here is an old idea[9]

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[9] S. F. Boys, *Proc. Roy. Soc., Ser. A*, **144**, 655 (1934), and references cited therein.

that was never fully tested, apparently because of the greater attention given to anisotropic group interactions as a result of Kirkwood's theory.[10] The present results suggest that the isotropic atom model has greater validity than is generally supposed. The model is attractive because reasonably good atom polarizabilities are available, the locations of the polarizable points are relatively well defined, and some polarizability is assigned to each atom rather than to a single point in a group of atoms. In regard to the values used here for the atom polarizabilities, it might be argued that these are not necessarily physically meaningful, since they are adjusted to fit $\bar{\alpha}$ values using a somewhat arbitrary model.[2] However, this fact does not guarantee that the same polarizabilities would be successful in accounting for optical rotations, since $\bar{\alpha}$ depends only on the diagonal elements of the $\mathbf{B}_{ij}$'s, while $[m]$ depends only on the off-diagonal elements. Thus, to the extent that the model is successful in predicting optical rotations, it is implied that the treatment of atoms as isotropically polarizable points is not too far from the truth. In the case of the CN group, this model might be doubted in particular because the valence electrons, which are primarily responsible for polarizability, are extensively shared between the atoms. Yet both the present results and those obtained previously on the polarizabilities of nitriles[3] do not suggest that there is any virtue in treating the CN group as a single point instead of two. In fact, there is less ambiguity in the sign of the rotation for the isotropic atom model.

The potential value of a theory such as this is in its use as a tool for determining molecular structures from observed rotations; one might at least hope to determine the absolute configuration from the sign of rotation, and in favorable cases information on conformation might be obtained. Enough atom polarizability data are available[2] to attempt this with a wide variety of molecules, the only additional data required being bond lengths and angles. In most cases care must be taken to test the sensitivity of the calculations to parameters that are not known accurately, as the case of MBCA illustrates.

Acknowledgment. This investigation was supported by a research grant from the National Institute of General Medical Sciences (GM-13684).

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Optical Rotations of Cyclohexanepolyols from Polarizability Theory

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Abstract: The optical rotations of the 22 possible optically active cyclohexanepolyols at 5893 Å have been calculated by the exact (all-order) version of polarizability theory, assigning isotropic polarizabilities to all atoms and, alternatively, single isotropic polarizabilities to the OH groups. Results are obtained for both the *C1* and *1C* chair forms. The signs agree with available experimental data when these are assigned to the chair form having the least axial substitution, but the predicted magnitudes are larger than observed in most cases. Rotations averaged over staggered OH conformations are comparable to those obtained by treatment of the OH group as an isotropic unit. Uncertainties due to propagation of errors in input parameters are mostly in the range 20–50%. The results correlate fairly well with the predictions of Whiffen's empirical rules, but large rotations are predicted in several cases where these rules predict zero rotation.

The optically active cyclohexanepolyols constitute an important class of compounds for testing a theory of optical rotation, since there are 22 possible enantiomorphic pairs in the series (excluding *gem*-diols) and many of these have been characterized experimentally as to absolute configuration and sodium D line (5893 Å) rotation.[1] In addition, the problem of conformational averaging is not insuperable in these compounds. This paper presents a study of the rotations calculated for all members of the series using the isotropic atom version of the polarizability theory.

Recent studies of the isotropic atom model have shown that the model is useful in accounting for polarizabilities of molecules,[2,3] and a few calculated rotations[4,5] have shown that it can give realistic results for this property as well. The present study was carried out as a further test of the model, both to gain insight into the mechanism of optical rotation and to explore the usefulness of the theory as a means of interpreting observed rotations in terms of absolute configurations or conformations.

The primary insight into the origin of optical rotation in the cyclohexanepolyols to date has come from the empirical rules of Whiffen,[6] which find impressive agreement with experiment in ascribing a rotational contribution to each pair of hydroxyls located on adjacent carbon atoms. This suggests that pairwise interactions of nearest neighbor hydroxyls dominate the rotation.[7]

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(1) T. Posternak, "The Cyclitols," Holden-Day, San Francisco, Calif., 1965.

(2) J. Applequist, J. R. Carl, and K.-K. Fung, *J. Amer. Chem. Soc.*, **94**, 2952 (1972).

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(7) See also the discussion by W. Kauzmann, F. B. Clough, and I. Tobias, *Tetrahedron*, **13**, 57 (1967).